3301. In all three cases, the answer is a single interval.

3302. Since all parabolae are stretches and translations of one another, you can prove this without loss of generality by using $y = x^2$.

Find the generic equation of a tangent line. Then set up versions at $x = a$ and $x = b$. Solve these simultaneously.

3303. Differentiate f and g. Set up a pair of inequalities for negative gradients and solve them individually. Then find the intersection of the solution sets.

3304. Consider a list of all $n!$ orders of the n objects. Then look at the overcounting factors.

3305. You can integrate this by inspection.

— Alternative Method —

For a longer way round, factorise the denominator and split the integrand into partial fractions.

3306. Use a circle of radius 3, so that the numbers work nicely. Calculate the areas of segments, via sectors and triangles.

3307. Use Δ to show that the first equation has no real roots. Find a fairly obvious root for the second equation. In the third equation, consider $\sin^2 x = -\tan x - 1$ and work graphically.

3308. Restrict the possibility space to the trial at which there is an end result.

3309. Substitute to produce a polynomial equation in a single variable, and factorise to show that it has a double root.

3310. The implication goes one way.

3311. (a) Differentiate with respect to either x or y.
(b) Consider the parity of the powers.
(c) Note that there are no other points at which the tangent is vertical/horizontal, and consider the behaviour as $y \to \pm\infty$.

3312. Draw the unit square $[0, 1] \times [0, 1]$ as the possibility space, and shade the successful region.

3313. Set up a definite integral, and integrate by parts (the parts are $u = \ln x$ and $\frac{dv}{dx} = 1$).

3314. The points equidistant from L_1 and L_2 lie on the angle bisectors of these lines. These are $x = a$ and $y = b$. The combined set of points can then be expressed as in the question.

3315. In each case, use a compound-angle formula, then a small-angle approximation.

3316. (a) Solve $y = 0$.
(b) Find the first and second derivatives.
(c) Show that $\frac{dy}{dx}$ is undefined at $(0, 0)$.
(d) Set up an inequality: the second derivative must be negative.
(e) Put the above information together with the behaviour as $x \to \pm\infty$.

3317. There are ${}^{10}\mathrm{C}_2 = 45$ ways of selecting two distinct integers. Work out how many of these differ by one from each other.

3318. (a) Explain why the various tensions could have arbitrarily large values, which aren't fixed by the information in the question.
(b) Take the equation of motion along the string.

3319. Differentiate using the quotient rule, and set the derivative to zero. Solve for k.

3320. Square both sides as they are, before rearranging to the form $\sqrt{\mathrm{f}(x)} = \mathrm{g}(x)$. Then square again. Note that squaring may introduce roots.

3321. The shortest distance between two curves lies along the normal to both. In the case of circles, this is the line of centres.

3322. This looks a bit complicated. But, once you have understood what is being asked, you don't have to do that much. You just need to show that there are infinitely many rational numbers x satisfying $\sin\left(x^2\right) > \frac{1}{2}$. Part-solve this inequality by finding any real interval I such that every $x \in I$ satisfies $\sin\left(x^2\right) > \frac{1}{2}$. Show that this has infinitely many rational numbers in it.

3323. Solve for the curve's x intercepts. Then either find the equation of the tangent at $x = 2.3$ and work out its x intercept, or use one iteration of Newton-Raphson to do the job for you.

3324. To be invertible on the domain $\mathbb{R}$, a cubic must have no turning points. This means it can have a maximum of one stationary point.

3325. Write the integrand as the sum of a constant and a proper fraction with a factorised denominator. At that point, you can integrate.

3326. This is true of all polynomials up to degree 3. Find a quartic counterexample.

3327. (a) The legs form a pyramid with an equilateral triangle E as the base. Find the distance from the vertices of E to the centre of E. Then use trig on a right-angled triangle to find $\sin\theta$.
(b) Consider that there are three thrusts holding the camera up. Resolve vertically to find $\cos\theta$ using the first Pythagorean trig identity.

3328. For large a, the triangle approaches equilateral, so cannot have an obtuse angle. The boundary case is one in which the largest angle is a right-angle. Analyse this case.

3329. The number of ways of choosing 2 squares from 9 is ${}^9\text{C}_2 = 36$. Out this possibility space, find the number of successful outcomes.

3330. (a) Draw a sketch, with e.g. $p = q = 1$. Also, rearrange the boundary equation $x^p y^q = 1$ to make y the subject.
(b) Carry out the integral in (a).
(c) Show that one of the limits is infinite.

3331. Consider the denominator in harmonic form, i.e. as $R\sin(2\theta + \alpha)$. You don't need to calculate α, only R.

3332. You can take f to be a positive polynomial wlog. Assume, for a contradiction, that f is a polynomial function of even degree with $\text{f}'(x) > 0$ for all x.

3333. Solve the boundary equation by factorising (don't multiply out the brackets). Also, sketch $y = x - a$ and $y = (x - a)^2$.

3334. Write the sequence in terms of the common ratio r. You need to show that $r^3 + 1 > r^2 + r$. Show that the boundary equation has no roots for $r > 1$, and therefore that LHS > RHS.

3335. Calculate the probability that $X = 1$ with and without the condition $XYZ = 6$, and compare.

3336. Put the log input over a common denominator. Then use a log rule to simplify.

3337. Eliminate the variables p and q, as if solving a set of simultaneous equations. Simplify as you go.

3338. Find the equation of the normal by differentiating implicitly with respect to x. Then solve a pair of simultaneous equations.

3339. (a) Draw a sketch of the (x, y) plane.
(b) Consider the formula for the area of a circle.
(c) Carry out the integral.

3340. Compare the magnitudes of the terms of ① to 1, and then compare the magnitudes of the terms of ① and ② to each other.

3341. Consider the set of x intercepts of each parabola. For the first set to be transformed to the second set by a reflection in $x = a$, the value a must be halfway between each x intercept and its image.

3342. A scale factor, due to the chain rule, is missing.

3343. Let $y = \arcsin x$. Then write $\tan y$ as $\frac{\sin y}{\cos y}$, and use a Pythagorean trig identity.

3344. The first branches are B and B'. Calculate their probabilities by adding two of the branches of the original diagram. Do likewise with the others: each branch marked A or A' needs a conditional probability.

3345. (a) Find the second derivative and factorise it.
(b) You need to show that the second derivative is zero at $x = 0$, but that it doesn't change sign. Consider the signs of the factors of the result in (a).

3346. Noting that an integral with respect to x calculates the signed area between a curve and the x axis, you can write down the answers to these without any calculation. Note the order of the limits in (b).

3347. Since the acceleration is vertical, you can calculate the resultant force by Pythagoras, without finding angles: the resultant of the two electromagnetic forces must act vertically.

— Alternative Method —

Call the angle between the right-hand force and the horizontal θ. Resolve horizontally, for which acceleration is zero, to find θ. At this point you can resolve vertically.

3348. Show that a specific value of x yields more than one y value on the curve: $x = 1$ is one such.

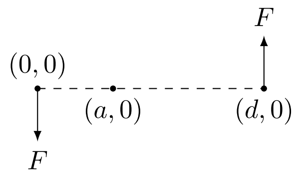
3349. The possibility space is an $m \times n$ rectangular grid. The set of successful outcomes is triangular.

3350. One of the statements is false. Consider the fact that, if integers differ by an odd number, then one of them must be even.

3351. (a) Consider the lengths of the sides.
(b) Find the lengths of the diagonals.

3352. Draw a clear sketch, and use the fact that radius and tangent are perpendicular.

3353. Use the following diagram:



Calculate the resultant moment around $(a, 0)$.

3354. Use a double-angle formula to combine the trig function, then integrate by parts with $u = 2x$.

3355. Sketch the region, and calculate the area of the two segments enclosed.

3356. Use a polynomial solver or numerical method to find the other intersection point. Then set up a single definite integral with respect to y.

3357. (a) Integrate term by term, noting that reversing the order of the limits negates the value of an integral.
(b) Use the substitution $u = 3x$.

3358. Use the generalised binomial expansion, and ignore terms beyond x^2.

3359. An iteration $x_{n+1} = \text{g}(x_n)$ has a fixed point iff $x = \text{g}(x)$. Set up such an equation, and show that it has at least two roots.

3360. There's no need to use a double-angle formula, as *both* trig functions take the input $2x$.

Instead, multiply up by $\sin 2x$. Then simplify the trig to make $\tan 2x$ the subject. Find the first four positive roots $2x = ...$, before dividing by 2.

3361. (a) Set up a $\left(1, 2, \sqrt{5}\right)$ right-angled triangle.
(b) The bowl is smooth, so the force exerted at each point of contact acts along the normal to the curve. Use part (a) once you have resolved vertically.

3362. Consider the equation $\text{f}(x) - \text{g}(x) = 0$, specifically the multiplicity of its roots.

3363. Start by expanding the brackets in $(x^2 + x + 1)^2$. Check the coefficients of x^4 and x^3, then work on the x^2 term.

3364. Using the cosine rule, set up a formula for c^2 in terms of r and θ. Differentiate this formula with respect to t.

3365. Consider the fact that $|x - y|$ must be positive. Hence, the other factor must also be positive.

3366. Expand $\left(1 - x^2\right)^{-1}$, with $-x^2$ playing the role of x in the generalised binomial expansion.

3367. Call the angle of projection θ. Find the values of θ which give $d = {}^{u^2}\!/_{2g}$. Consider the set of successful values out of the possibility space $[0, 90°]$.

3368. Integrate by inspection, considering the integrand as the result of differentiation by the chain rule.

3369. Find the equation of the normal at $x = p$.

3370. You just need a way of listing clearly. One way is to classify by the side lengths of the triangles.

3371. (a) Multiply the probability of picking the first two correctly (CC) and the third incorrectly (I) by the number of orders of CCI.
(b) Multiply the probability by 5.
(c) Use another binomial probability.
(d) Use the same method as in (b).

3372. Assume, for a contradiction, that $n + 1$ distinct points on a polynomial graph of degree n are collinear. Let $y = mx + c$ be the equation of the line. Then consider the equation $\text{f}(x) = mx + c$.

3373. The relevant facts are the smoothness of the peg and the lightness of the string. Explain how you know that the tension is the same throughout the string, and also how you know that the magnitude of an external force applied to a light string must be equal to the tension.

3374. Only one of these is true.

3375. Write the ellipse as $\left(\sqrt{2}x\right)^2 + \left(\sqrt{5}y\right)^2 = 10$, and consider stretches of $x^2 + y^2 = 10$ in the x and y directions.

3376. Use a Pythagorean identity on the LHS and also a double-angle identity on the RHS.

3377. (a) Use a vertical *suvat* to find the time of flight, then consider the (x, y) horizontal.
(b) Consider the symmetry of the parabolic flight.
(c) Again consider symmetry. Find the angle of projection.

3378. (a) Differentiate implicitly with respect to y.
(b) Substitute $y = 0$.
(c) Differentiate again with respect to y, and sub in both $y = 0$ and $\frac{dy}{dx} = 0$. To show a sign change, consider the signs of the factors.
(d) Put the information above together.

3379. The LHS is a geometric series.

3380. The various triangles are equilateral. Use the fact that the centre of an equilateral triangle divides its height in the ratio 1 : 2.

3381. Assume, for a contradiction, that $y = \mathrm{f}(x)$ is both concave and increasing on $(0, \infty)$, and stationary at the origin.

Consider the gradients at $x = 0$ and e.g. $x = 1$.

3382. The graph is broadly akin to $y = \cos x$, except for the fact that the single roots have been converted into triple roots.

3383. In each case, find the probability of one specific successful outcomes, then multiply by the number of different versions of that successful outcome.

3384. Fixed points of $x_{n+1} = \mathrm{g}(x_n)$ are roots of $x = g(x)$.

3385. Different techniques are required in each:

① Use the discriminant.
② The equation isn't a quadratic.
③ Consider the sign of the two roots for $\sqrt{x}$.

3386. (a) The assumption concerns the string itself.

(b) Model the two static blocks as a single object, and assume that the friction between them is limiting.

(c) The formula in (b) breaks down for $\mu = 1$. This isn't an error, and points to the question in (b) not making sense for $\mu = 1$.

3387. Assume, for a contradiction, that the shortest path PQ is not normal to curve C at point P. Then show that moving point P in a certain direction must reduce the distance $|PQ|$.

3388. (a) Differentiate by the quotient rule.

(b) Remember that, having substituted in, the two sides of the DE must be *identically* equal.

3389. Translate the given information into one equation for k, by equating the differences.

3390. Find the equation of a generic tangent to the curve at $x = p$. Show that its y intercept is positive.

3391. Consider the scenario with a smooth pulley. Then consider the change when friction is added.

3392. Multiply out the brackets. Each term is then a standard derivative.

3393. Use the formula ${}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$.

3394. (a) Set $y = 0$.

(b) Consider the sign of the numerator and of the denominator.

(c) Differentiate by the quotient rule.

(d) Put all of the above together.

3395. Call the people $A_1, A_2, B_1, B_2, ...$ Place A_1 wlog, then find the probability that A_2 sits opposite. Then place B_1, and so on.

3396. (a) Set up horizontal and vertical *suvat*s, and eliminate t.

(b) Solve the equation using a Pythagorean trig identity. Choose the appropriate result.

3397. Consider writing $x = X + k \times 10^{-m}$, where k is the mth digit.

3398. Consider the side length of the resulting square.

3399. Because the graphs are standard shapes, this is easiest if you convert to Cartesian equations first.

3400. Overall, the integrand is a product. You'll also need the chain rule to differentiate the factors.

——— End of 34th Hundred ———